PHY 153 - Final Project Report

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1 Project Description

This project featured the use of the photoelectric effect to calculate and compare the Work Functions (W) of various cathode materials (Sodium, Platinum, Silver, Potassium, and Cesium), as well as Planck's Constant (h). This was accomplished with the formula:

$$K_{max} = h\nu - W \tag{1}$$

where ν is the frequency of the light incident on the cathode material and K_{max} is the maximum kinetic energy of the electron emitted.

For each cathode material, the true Work Function of the material (W_{True}) in terms of eV, an array of frequency measurements (ν) in terms of 10^{14} Hz, and an array of equal length of K_{max} measurements in terms of eV were provided. The uncertainty for all of the K_{max} measurements was taken to be 1.0 eV. With this data, the following tasks were to be completed:

1. Plotting the K_{max} measurements versus the frequency measurements (ν) for each of the five cathode materials.

2. Using Eq. (1) to plot a line of best fit for each plot.

3. Determining the Work Function (W) and Planck Constant (h) measurements for each of the five cathode materials. (It should be noted here that the Work Function and Planck Constant are β_0 and β_1, respectively, for the fitting parameters of the fitted line of each graph.)

4. Quantifying our results using the χ^2 (S_m) calculation and from this value, calculating the p-value for each of the five cathode materials.

5. Comparing our measured Work Function value with the true Work Function of each of the five cathode materials using the z-score and the p-value derived from that z-score.

6. Determining the best measurement of Planck's Constant (h) from the five cathode materials and comparing it to the true value of Planck's Constant using the z-score and the p-value derived from that z-score.

2 Data Analysis

2.1 Question 1: Part A

In order to judge the quality of the results, one must look at the p-values shown in Table 1. Simply put, a p-value is a measure of how well the data points are fitted to the fitted line, where a higher p-value indicates a stronger fitting. In this project, a p-value can be calculated in one of two ways. In the other two parts, the p-value will be derived from a z-score, which will be explained in its relevant section. However, for this part of the project, the p-value is calculated using the S_m value, also referred to as the χ^2 (chi2) method. This value is calculated according to the following formula:

$$S_m = \sum_{i=1}^{N} \left(\frac{K_{max,i} - \beta_0 - \beta_1 \nu_i}{\sigma_{K_{max},i}} \right)^2 \tag{2}$$

This S_m value is then used in the scipy.stats function "chi2.cdf()", which takes two parameters, S_m and k, where k is the number of free parameters. This is equivalent to the number of data points minus 2, since β_0 and β_1, are two degrees of freedom that are already defined as the Work Function and Planck Constant, respectively, of the fitted line. The p-value is then calculated as 1.0 - scipy.stats.chi2.cdf(S_m, k)

Looking at the p-values of the five cathode materials, Platinum is by far the most apparent outlier, with a p-value of less than 4%. This is an incredibly low value, and it suggests that the data points for Platinum lie further away from its fitted line than any of the other cathode materials. Silver also suggests a weaker relationship between the data and the fitted line, but not as weak as Platinum. Sodium, Potassium, and Cesium all have relatively high p-values, suggesting that these are all good fits.

These relationships are supported by observing the graphs. In particular, observing how far away the data points fall from the fitted line. For example, Sodium has the highest p-value of the five cathode materials, and its data points fall in a relatively straight, tight line along the fitted line. Comparing this to Platinum, whose data points spread out more and don't follow as linear a pattern, it is apparent that the fitted line for Sodium is much better than that of Platinum. Potassium and Cesium both have data points arranged in a similar fashion to Sodium, and all three have similar p-values. Silver's data points are not quite as spread out as Platinum's, but also are not fitted as tightly as the other three. As expected, Silver's p-value falls right in the middle of Platinum's p-value and the other three's p-values.

2.2 Question 1: Part B

In order to judge how well our Work Function results compare to the expected values, a z-score will be calculated, and from this z-score, a p-value will be found. A z-score signifies how far away the data falls from the expected value in terms of standard deviations, where a smaller z-score means fewer standard deviations and thus, a better match to the expected value. The z-score can be calculated as follows:

$$z = \frac{x - x_{True}}{\sigma_x} \tag{3}$$

As mentioned previously, a p-value can also be calculated from a z-score. It requires the use of the scipy.stats function "norm.cdf()", which takes one parameter, the z-score. The p-value is then calculated as 2.0 * scipy.stats.norm.cdf(z).

Using this method, it is found that Sodium is the outlier in terms of Work Function, as it is the only one of the cathode materials with a z-score of a magnitude higher than 1.0, nearly 2.0. This is to say that the result for Sodium's Work Function falls farther away from its expected value than the other cathode materials do from theirs. A z-score of 2.0 means that the results fall two standard deviations away from the expected value, which is equivalent to a 5% "match", which is not a good match at all. It is apparent that Sodium is not a good fit. Calculating the p-value from the z-score reveals that Sodium also has a significantly lower p-value compared to the other four cathode materials. It can also be noted that the lower the p-value is, the higher the z-score is and, naturally, the farther away the calculated Work Function was from the true value. It is sufficient to say that for the calculation of Work Function in this section, Sodium's results do not produce a sufficient approximation. As for the other four materials, two of them (Platinum and Potassium), have relatively high p-values and low z-scores, whereas the other two (Silver and Cesium) have a p-value and z-score between the other groups. The approximations for Platinum and Potassium are clearly reasonable approximations. In the case of Silver and Cesium, however, it is less apparent how good their approximations are.

Another way to interpret this data without p-values or z-scores is to simply look at the results and uncertainties. For each cathode material, the calculated Work Function was lower than the true value, but each measurement has an uncertainty to it. If the uncertainty of each material's Work Function is added to its respective values (that is, in the case of Sodium, 1.36 eV + 0.50 eV = 1.86 eV), it is found that all of the Work Functions can reach their respective true value when factoring in uncertainty, with the exception of Sodium. The results of the other four cathode materials are all close enough to their true value that the uncertainty is able to cover the gap, but Sodium's value is such a poor approximation that this is not possible. As such, it can be concluded that, as previously mentioned, Sodium's approximation is not appropriate, whereas Platinum's and Potassium's are. However, if the results are interpreted in this manner, it can also be concluded that Silver's and Cesium's approximations are also acceptable, albeit to a weaker degree.

2.3 Question 2

Question 2 asked to use the five values of Planck's Constant (h) from Part 1 to find the best value on h, and then use the z-score and p-value to justify whether it was a good approximation.

One way to go about doing this is to calculate the average Planck Constant for all five materials. Doing this produces an Average Planck Constant of 0.399 ± 0.014 eV. Comparing this to the true value of the Planck Constant, 0.4135667696 eV, the resulting z-score is -1.007, and the resulting p-value is 0.3137 (31.37%)

With a z-score of approximately -1.0, meaning the value is approximately 1 standard deviation away from the expected value, the average is a reasonable approximation for the value of the Planck Constant. However, it should be noted that a much better estimate for the Planck Constant already exists within the individual cathode material measurements. The Planck Constant, z-score, and p-value of each material are listed in Table (4).

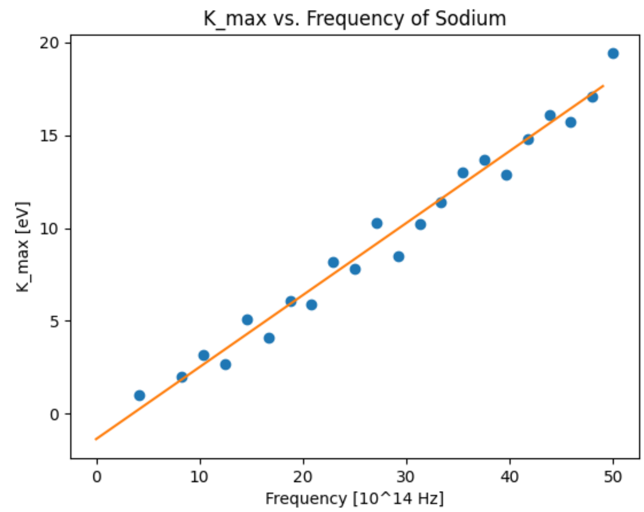
Looking at the table, it is apparent that the average value calculated previously is skewed due to Sodium's contribution, whose measurement is much farther from the true value than the others. While the Average Planck Constant's measurement is a reasonable approximation of the Planck Constant, it is not the best approximation of the Planck Constant that can be derived from this project. If a better result is desired, one should disregard Sodium's contributions to the average calculation. However, considering that all of the measurements fall below the true value, the average may not be the best approximation of the Planck Constant in the first place, a notion supported by the fact that, when adding the uncertainty to the average Planck Constant, the true value is not reached. Instead, it should be noted that the Planck Constants that were calculated by Potassium and Cesium are much closer to the true value than the average. As such, it can be argued that one of these values should be taken to be the "best" value of the Planck Constant instead of the Average Planck Constant.

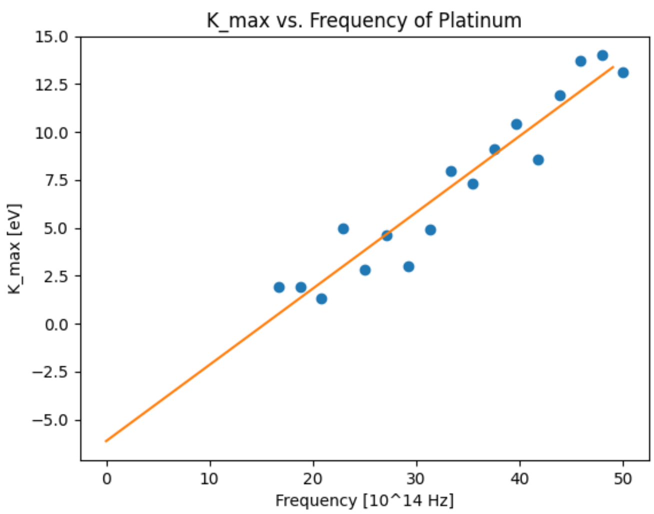
Furthermore, it should be noted that, much like the Work Function values described in the previous part, adding the uncertainty to the Planck Constant values reveals that all of the cathode materials except for Sodium can attain the true value of the Planck Constant.

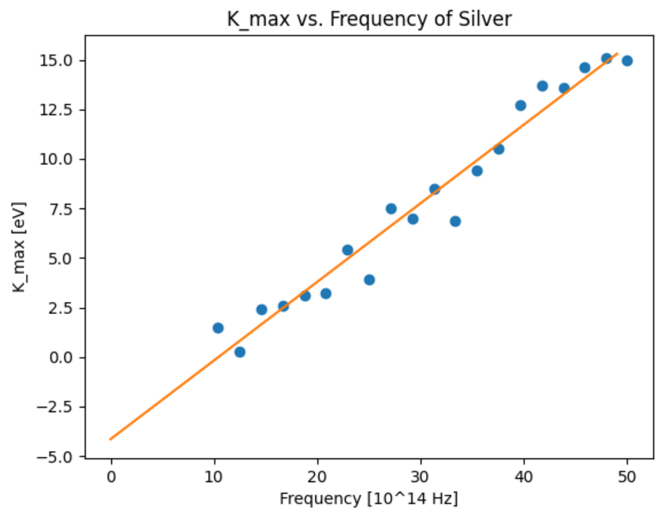
3 Results

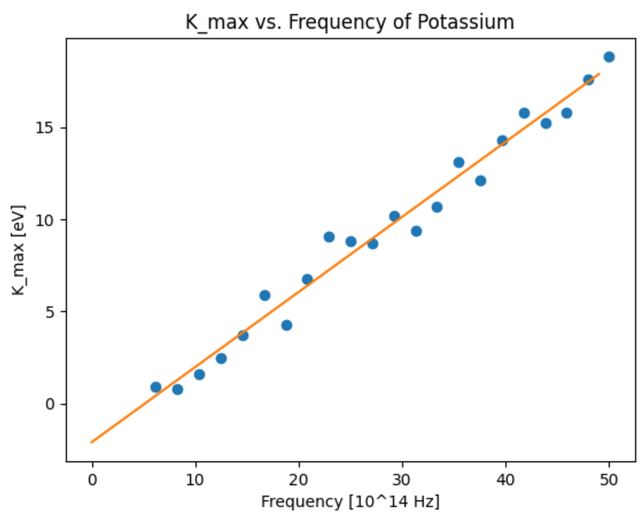
3.0.1 Question 1: Part A

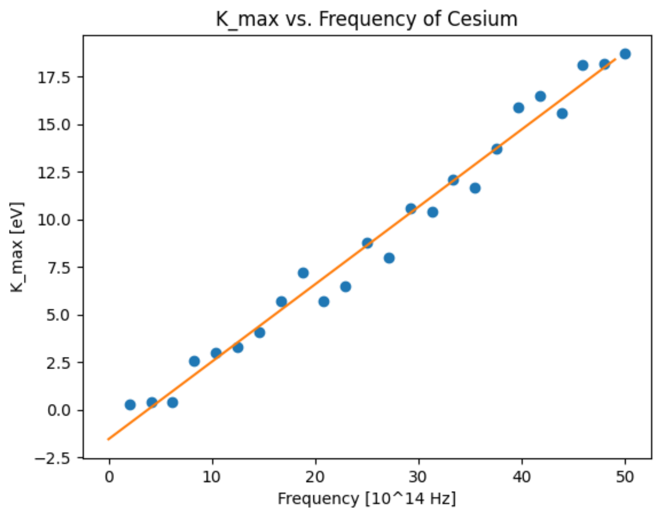
The results from this section are displayed below. Each graph shows the plotted K_{max} measurements versus the frequency measurements (ν). The fitted line is of the form described by Eq. (1), where W is β_0 and h is β_1:






K_max vs. Frequency of Silver
K_max [eV]
Frequency [10^14 Hz]


K_max vs. Frequency of Potassium
K_max [eV]
Frequency [10^14 Hz]



Below is a table containing the relevant data for this part. For each cathode material, the values found for the Planck Constant (h) and Work Function (W), as well as their uncertainties, are listed, as well as the $\chi^2(S_m)$ value and the corresponding p-value:

Cathode Material	h (eV)	W (eV)	χ^2 (S_m)	p-value
Sodium	0.388 ± 0.016	1.36 ± 0.50	12.850	0.8837 (88.37%)
Platinum	0.398 ± 0.024	6.13 ± 0.83	26.164	0.0363 (03.63%)
Silver	0.397 ± 0.019	4.14 ± 0.61	17.763	0.4714 (47.14%)
Potassium	0.407 ± 0.016	2.08 ± 0.50	14.446	0.8072 (80.72%)
Cesium	0.407 ± 0.014	1.53 ± 0.42	15.469	0.8413 (84.13%)

Table 1: Fitting Parameters and Data Collected - Question 1: Part A

3.0.2 Question 1: Part B

Below is a table containing the relevant data for this part. For each cathode material, the true Work Function (W_{True}) and measured Work Function (W) with uncertainty are listed, as well as the corresponding z-score and p-value for the Work Function:

Cathode Material	W_{True} (eV)	W (eV)	z-score	p-value
Sodium	2.3	1.36 ± 0.50	-1.892	0.0584 (05.84%)
Platinum	6.4	6.13 ± 0.83	-0.327	0.7439 (74.39%)
Silver	4.7	4.14 ± 0.61	-0.920	0.3578 (35.78%)
Potassium	2.2	2.08 ± 0.50	-0.242	0.8090 (80.90%)
Cesium	1.9	1.53 ± 0.42	-0.870	0.3843 (38.43%)

Table 2: Work Function Results and Data Collected - Question 1: Part B

3.0.3 Question 2

Below is a table containing data for the average value of the Planck Constants found in Question 1: Part A. It features the Average Planck Constant value, the true value of the Planck Constant, the z-score, and the p-value.

Avg. Planck Constant (eV)	h_{True} (eV)	z-score	p-value
0.399 ± 0.014	0.4135667696	-1.007	0.3137 (31.37%)

Table 3: Data for the Average Planck Constant - Question 2

Below is a table containing data for each cathode material's value of the Planck Constant, as well as their respective z-scores and p-values.

Cathode Material	h(eV)	z-score	p-value
Sodium	0.388 ± 0.016	-1.617	0.1058 (10.58%)
Platinum	0.398 ± 0.024	-0.647	0.5179 (51.79%)
Silver	0.397 ± 0.019	-0.902	0.3669 (36.69%)
Potassium	0.407 ± 0.016	-0.409	0.6826 (68.26%)
Cesium	0.407 ± 0.014	-0.477	0.6334 (63.34%)

Table 4: Individual Material's Planck Constants and Data - Question 2

4 Conclusion

This project involved comparing Work Functions (W) and Planck Constants (h) calculated from a data set of Sodium, Platinum, Silver, Potassium, and Cesium frequencies and electron kinetic energies to their expected values. This was accomplished through calculations of $\chi^2(S_m)$, z-scores, and p-values. A summary of the data collected for the Work Function and Planck Constant can be found in the tables below:

Cathode Material	W_{True} (eV)	W (eV)	z-score	p-value
Sodium	2.3	1.36 ± 0.50	-1.892	0.0584 (05.84%)
Platinum	6.4	6.13 ± 0.83	-0.327	0.7439 (74.39%)
Silver	4.7	4.14 ± 0.61	-0.920	0.3578 (35.78%)
Potassium	2.2	2.08 ± 0.50	-0.242	0.8090 (80.90%)
Cesium	1.9	1.53 ± 0.42	-0.870	0.3843 (38.43%)

Table 5: Summary of Work Function Data

Cathode Material	h(eV)	z-score	p-value
Sodium	0.388 ± 0.016	-1.617	0.1058 (10.58%)
Platinum	0.398 ± 0.024	-0.647	0.5179 (51.79%)
Silver	0.397 ± 0.019	-0.902	0.3669 (36.69%)
Potassium	0.407 ± 0.016	-0.409	0.6826 (68.26%)
Cesium	0.407 ± 0.014	-0.477	0.6334 (63.34%)
Average	0.399 ± 0.014	-1.007	0.3137 (31.37%)

Table 6: Summary of Planck Constant Data (True $h = 0.4135667696$ eV)

In order to determine whether or not the results match the theory, one must look to the two left-most columns of both tables. The z-score and p-value indicate how well the data fits the expected values. A lower z-score and a higher p-value indicate that the results match the theory.

Of the five cathode materials, the material with the best fitting data is Potassium, as it has the lowest z-score and highest p-value for both the Work Function and Planck Constant. There is no doubt that Potassium's results match its theory.

For Platinum, the results are split. The Work Function's z-score and p-value suggest that that results match the theory confidently. However, it is not clear whether or not the Planck Constant's result matches the theory. A p-value of roughly 52% does not show ultimate confidence. However, it can be argued that, since it is higher than 50%, the results are more favorable than unfavorable. As such, it can be claimed that the Planck Constant's result does match the theory, albeit rather weakly.

On the contrary, it is apparent that the Work Function and Planck Constant values for Sodium do not fit well at all. Sodium has the highest z-score and lowest p-value of the five materials for both Work Function and Planck Constant. With both p-values at or below 10% and both z-scores move than 1.5 standard deviations away from the expected value, it can be confidently said that the Work Function and Planck Constant values for Sodium do not match the theory.

The same can be said of Silver, whose high z-scores and low p-values suggest that the results are not a very confident match to the theory. It should be said, however, that the results of Silver do match slightly better than that of Sodium, as it has both z-scores below (though very close to) 1.0 standard deviation away from the expected value. Furthermore, its p-values are both around 35%. They are, however, confidently below 50%, so it is safe to say that the results of Silver do not match the theory.

Cesium is by far the most interesting of the five cathode materials. The z-score and p-value of its Work Function strongly indicate that the results do not match the theory, yet its Planck Constant's indicate that the results do match the theory. This is made even more interesting by the fact that its data points, as described in Question 1: Part A, match its fitted line rather strongly. Despite this, it must be concluded that, while the Planck Constant results match the theory, the Work Function results do not.

There are a few things that can be done in order to improve future measurements. The first and most obvious is to collect more data points. The more data there is to evaluate, the more likely it is that the results will match the theory more closely, assuming the data is collected properly. In addition to this, recording uncertainty for the frequency data would aid in getting results to reach their expected values. Including uncertainty allows more leeway in the results matching up, so it is recommended to include as much uncertainty as possible in calculation. One final thing to do is collect multiple data sets in a variety of setups. There may be some sort of external interference on the system when performing the experiment once, so performing it multiple times with different setups will help to reduce the effects of any external interference.

5 Appendix

The code used to produce these figures and values has been submitted to zyBooks via zyLabs 14.8 and attached via e-mail along with this LaTeX file.